# Error estimates for spectral semidiscretizations of several cases of asymptotic models

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**Abstract**

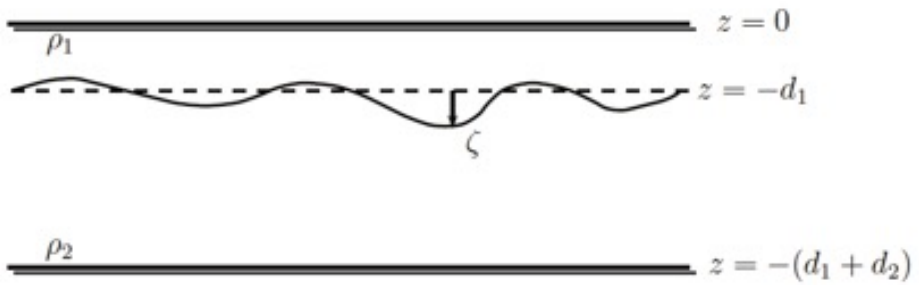


Figure 1:

The idealized model we are interested in consists of an ideal fluid (inviscid, homogeneous, irrotational, incompressible ) in a two-layer system with different densities $\rho_1 < \rho_2$, and depths $d_i, i = 1, 2$. A simple, one-dimensional representation is sketched in Fig. 1. Our main point of interest is the evolution of the function $\zeta(x, t)$, which represents the deviation of the interface of the two layers from its rest position, as well as the velocities of the fluid.

Among the different asymptotic models derived, the present talk will focus on the numerical approximation to the one-dimensional version of the PDE systems corresponding to three regimes:

1. Boussinesq/Boussinesq (B/B). In this case, a Boussinesq regime is assumed in both layers. This means that $\delta \approx 1$, and that the dispersive and nonlinear effects are assumed to be small and of comparable size to both fluid domains, that is
$$\mu \sim \mu_2 \sim \epsilon \sim \epsilon_2 \ll 1. \tag{1}$$
2. Boussinesq/Full Dispersion (B/FD). This regime assumes that:
   (a) The deformations are of small amplitude for both layers ($\epsilon \ll 1, \epsilon_2 \ll 1$).
   (b) A Boussinesq regime for the upper layer (dispersive and nonlinear effects of the same order of magnitude) and the lower layer is not necessarily shallow; that means
   $$\mu \sim \epsilon \ll 1, \quad \mu_2 \sim 1,$$
   for which we have that $\delta^2 \sim \epsilon$ and $\epsilon_2 \sim \epsilon^{3/2} \ll 1$.
3. Intermediate Long Wave (ILW) and Benjamin-Ono (BO). In both regimes, the upper layer is assumed to be shallow ($\mu \ll 1$) while for the lower layer the deformations are assumed to be of small amplitude ($\epsilon_2 \ll 1$). In addition:
   (a) In the ILW regime, the deformations with respect to the upper layer are assumed to be small, with
   $$\mu \sim \epsilon^2 \sim \epsilon_2 \ll 1, \mu_2 \sim 1.$$

(b) The BO regime corresponds to the limiting case

$$\mu \sim \epsilon^2 \sim \epsilon_2 \ll 1, \mu_2 \sim \infty.$$

This talk is mainly focused on the spatial discretization of the periodic ivp's with spectral methods.