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Título: Extending the Gibbon-Fokas-Doering stagnation-point-type ansatz to finite-energy initial conditions: A solution to the Navier-Stokes Millennium Prize Problem?

Abstract: The stagnation-point-type solution to the 3D incompressible Navier-Stokes equations found in [Gibbon, Fokas and Doering, Physica D 132, 497 (1999)] produced an infinite family of solutions to the 3D incompressible Euler equations that blow up in a finite time. There is an exact formula for the singularity time as a functional of the initial conditions [Constantin, Int. Math. Res. Not. 2000, 455 (2000); Mulungye, Lucas and Bustamante, J. Fluid Mech. 771, 468 (2015); J. Fluid Mech. 788, R3 (2016)], and the solutions to this and related models are best understood in terms of infinitesimal Lie symmetries [Bustamante, Phil. Trans. R. Soc. A 380, 20210050 (2022)]. The main drawback of these solutions, from the viewpoint of the Clay Millennium Prize, is that the velocity field depends linearly on the out-of-plane spatial coordinate, and thus the initial condition has infinite energy. In this talk, I will present a way to extend these solutions to have an arbitrary dependence on the out-of-plane coordinate, allowing in principle for finite-energy solutions. This extension seems to break the infinitesimal Lie symmetry structure inherent to the previous infinite-energy solutions, so a statement regarding finite-time blowup is not yet available analytically in the finite-energy case. However, the extension allows for a novel numerical attempt at the finite-energy solution, via a hierarchy of systems of coupled 2D partial differential equations, which are much easier to handle than a full 3D problem. I will present results and prospects and discuss potential applications to real-life experiments.