The $2D$ nonlinear shallow water equations with a partially immersed obstacle

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This talk is devoted to the proof of the well-posedness of a model describing waves propagating in shallow water in horizontal dimension d = 2 and in the presence of a fixed partially immersed object. We first show that this wave-interaction problem reduces to an initial boundary value problem for the nonlinear shallow water equations in an exterior domain, with boundary conditions that are fully nonlinear and nonlocal in space and time. This hyperbolic initial boundary value problem is characteristic, does not satisfy the constant rank assumption on the boundary matrix, and the boundary conditions do not satisfy any standard form of dissipativity. Our main result is the well-posedness of this system for irrotational data and at the quasilinear regularity threshold. In order to prove this, we introduce a new notion of weak dissipativity, that holds only after integration in time and space. This weak dissipativity allows high order energy estimates without derivative loss; the analysis is carried out for a class of linear non-characteristic hyperbolic systems, as well as for a class of characteristic systems that satisfy an algebraic structural property that allows us to define a generalized vorticity. We then show, using a change of unknown, that is possible to transform the linearized wave-interaction system into a non-characteristic system that satisfies this structural property and for which the boundary conditions are weakly dissipative. We can therefore use our general analysis to derive linear, and then nonlinear, a priori energy estimates. Existence for the linearized problem is obtained by a regularization procedure that makes the problem non-characteristic and strictly dissipative, and by the approximation of the data by more regular data satisfying higher order compatibility conditions for the regularized problem. Due to the fully nonlinear nature of the boundary conditions, it is also necessary to implement a quasilinearization procedure. Finally, we have to lower the standard requirements on the regularity of the coefficients of the operator in the linear estimates to be able to reach the quasilinear regularity threshold in the nonlinear well-posedness result.

Reference: T. Iguchi, D. Lannes, *The* $2D$ *nonlinear shallow water equations with a partially immersed obstacle*, 2023.